Visualization of Topology with Uncertainty

Using Discrete Flow Maps

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with

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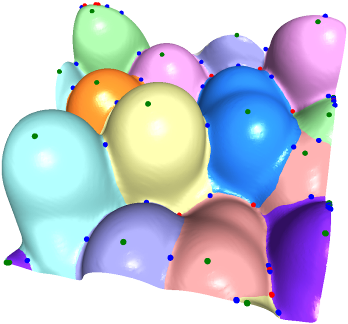
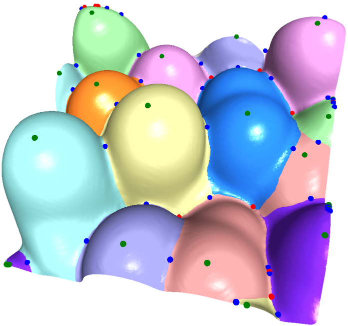
A Need for Stable Topological Structures

- Topological structures can provide meaningful analysis to encode features of data.



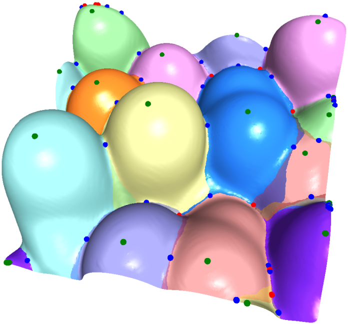
A Need for Stable Topological Structures

- Topological structures can provide meaningful analysis to encode features of data.
- Inherent unknowns such as noise and uncertainty mean interpreting the data is not a cut-and-dry process.

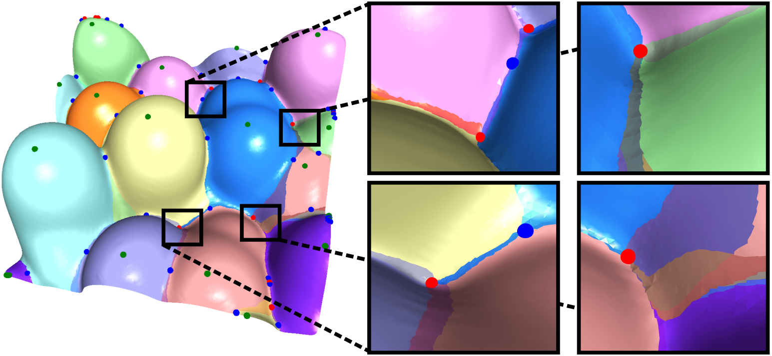


A Need for Stable Topological Structures

- Topological structures can provide meaningful analysis to encode features of data.
- Inherent unknowns such as noise and uncertainty mean interpreting the data is not a cut-and-dry process.
- Instead, these instabilities in the computation should be explicitly tracked and represented in the output.



Rayleigh-Taylor Instability, Close Up



Project Summary

Problem

Compute topological structures on vector-valued data robustly.
Show instabilities explicitly to provide more complete view of flow.

Solution

A new data structure for representing flow, *discrete flow maps*.

Impact

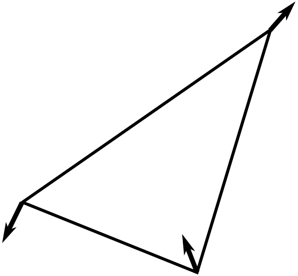
New views of flow fields that improve the quantification of error, complement traditional flow visualization techniques, and are a first step forward for studying topological segmentations when the data is uncertain.

Representing Vector Fields

- Currently, vector fields are almost exclusively represented as sampled vectors with values interpolated at unsampled regions
- To perform analysis and visualization, integration-based techniques are commonly used.
- These can suffer from compound numeric errors, leading to inconsistencies and instability.
- Two problems:
  1. No control over the stability of computation, and
  2. End-user has no quantification of where/what error has occurred.

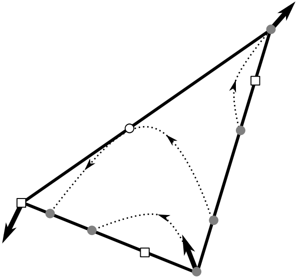
A New Representation

- Let $\mathbf{V} : \mathcal{M} \to \mathbb{R}^2$ be a 2-dimensional vector field on a 2-manifold $\mathcal{M}$.
- Commonly, $\mathcal{M}$ is discretized as a triangulation T.
- $\mathbf{V}$ is discretized by storing a vector at each vertex of T.
- The vectors within triangles are interpolated from the corners.



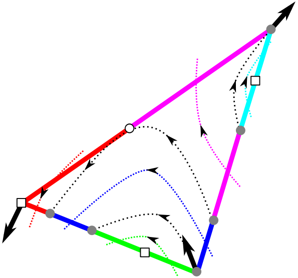
Flow

- $\mathbf{V}$ imparts a flow ϕ, a parametric path that a massless particle experiences as it travels with instantaneous velocity $\mathbf{V}$.
- $\frac{d\phi(t,x)}{dt} = \mathbf{V}(x)$, where $\phi(0,x) = x$.
- Flow could be computed by solving this differential equation explicitly, instead integration techniques are commonly used.
- Flow transitions at various boundary points.

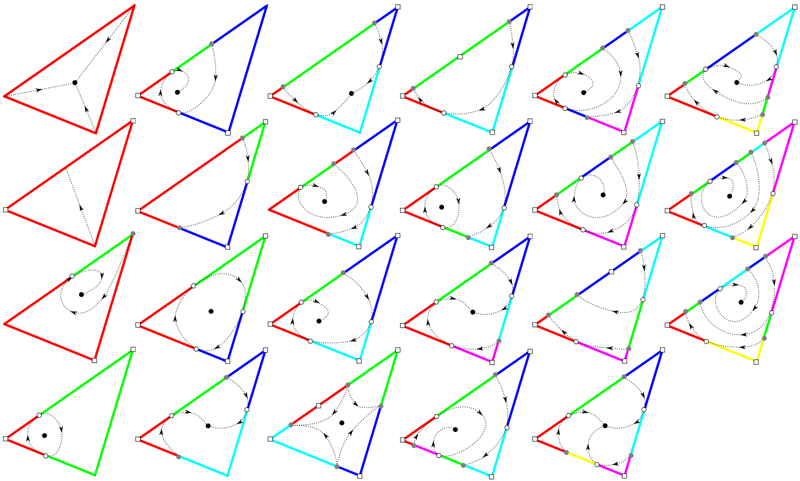


Encoding Vector Field Data with Flow Maps

- Let Δ be a triangle in T.
- The forward flow map, $\xi^+ : \mathrm{bd}\,\Delta \to \Delta$, maps points on the boundary to where they exit under the flow.
- If there is a critical point, the flow may never exit.
- The backward flow map, $\xi^- : \mathrm{bd}\,\Delta \to \Delta$ describes reverse flow behavior for boundary points with outward vectors.
- ξ^+ and ξ^- completely describe the behavior of the flow through Δ.

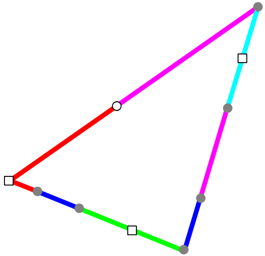


Summary: The 23 Topological Cases for Maps



Encoding Maps

- To encode a map, we simply need to store a *link* describing each pair of boundary intervals which are mapped to each other.
- In the case of a critical point, intervals can be mapped directly to the critical point.
- We can discard the vectors and flow paths through.
- Because there are a limited number of topological cases, we know that there can only be a small number of links



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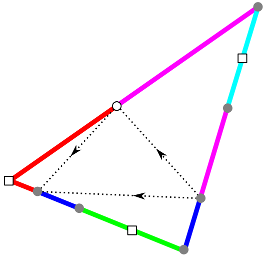
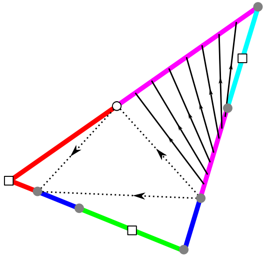
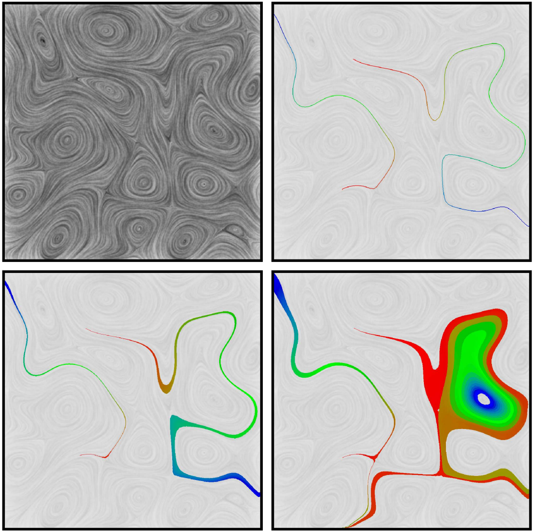


Replacing Integration with Map Lookup

- The power of the flow maps is they are a precomputed form of integration across a triangle.
- Thus, we can replace an integration step with a lookup through a map
- Map composition allows us to form a new concept of a streamline.
- Computationally, this lookup is simply a linear interpolation across the link.



Demo - Visualization Comparisons

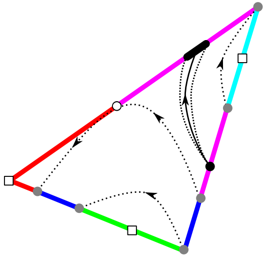


Topological Skeletons

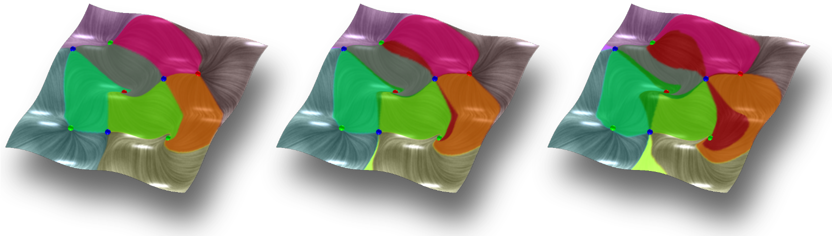
- Topological skeletons [Helman and Hesselink, 1989] and other decompositions have become a standard technique for visualization and analysis of vector fields.
- These techniques grow a four separatrices (2 forward, 2 backward) from each saddle to their terminal sinks and sources.
- The resulting structure partitions the domain into cells where all flow starts from a common source and ends up at a common sink.
- If the flow is irrotational, it is the Morse-Smale complex.
- When the flow has rotation, additional computations are needed to decompose the field, handle closed orbits, etc.

Adding Uncertainty to the Maps

- When computing separatices and closed orbits, compound numerical error can cause inconsistencies.
- Our representation naturally preserves source to destination correspondence of flow.
- In addition, we can model error or uncertainty in the destination by mapping a points to intervals.
- This leads to the concept of a *stream-front*, which goes from one source to many destinations



Demo - Fuzzy Topology



Future Research Avenues

- Time can be incorporated into the maps (although without time the topology is the same).
- Including a more sophisticated model for quantifying uncertainty and error.
- Multiple flow maps could be represented in a single triangle.
- Extending the same ideas to 3D flow.

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